

## Chapter 9

# Subtraction

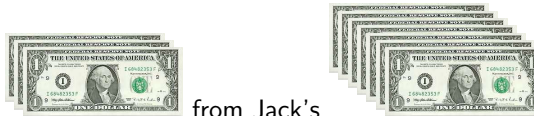
Detaching A Collection From Another, 1 – Language For Subtraction, 2 – Procedure For Subtracting A Number-Phrase, 4 – Subtraction As Correction, 5.

### 9.1 Detaching A Collection From Another

Given two collections, the third fundamental issue is to **detach** the *second* collection from the *first* collection. This is the second instance of an *operation*.

The real-world *process* is to mark off the items of the first collection that are also in the second collection and to look at all the unmarked items as making up a single collection that we shall also call the **resulting collection**.

**EXAMPLE 1.** To detach Jill's



from Jack's

- i. We set Jill's collection to the right of Jack's collection



- ii. We mark off the items in Jack's collection that are also in Jill's collection

—  
—  
minus  
bar



- iii. The unmarked items in the first collection make up the *resulting collection*



## 9.2 Language For Subtraction

In order to represent on paper the result of an *operation*, such as *detaching* a second collection from a first collection, we need to expand again our mathematical language but we will proceed in essentially the same manner as we did with the language for *addition*.

1. The first thing we need is a symbol, called *operator*, to represent the *operation*. In the case of *detaching* a second collection from a first collection, we will of course use the *operator*  $-$ , read as “minus”.

To represent on paper the result of *detaching* a second collection from a first collection, we will of course use the *operator*  $-$  read **minus**.

Here again, just as with the symbol  $+$ , this use of the symbol  $-$  is only one among very many different uses of the symbol  $-$  and that this will create in turn many difficulties. We shall deal with these difficulties one at a time, as we encounter each new use of the symbol  $-$ .

**NOTE.** It should be stated right away, though, that this use of the symbol  $-$  is only one among very many different uses of the symbol  $-$  and that this will create in turn many difficulties. We shall deal with these difficulties one at a time, as we encounter each new use of the symbol  $-$ .

2. Given two collections represented by number-phrases, we will represent *detaching* the second collection from the first by a *specifying-phrase* that we write as follows:

- i. We write the first number phrase:

first number phrase

- ii. We write the symbol for *subtracting*:

first number phrase  $-$

- iii. We write the second number-phrase over the **bar**:

first number phrase  $-$  second number phrase

Altogether then, the specifying phrase that corresponds to *detaching* from a first collection a second collection is:

first number phrase  $\underline{\hspace{2cm}}$  — second number phrase

identify  
identification-sentence  
arrowhead  
=

**EXAMPLE 2.** In order to say that we want to *subtract* from the first number-phrase 5 Washingtons the second number-phrase 3 Washingtons we write the *specifying phrase*:

$$5 \text{ Washingtons } \underline{\hspace{2cm}} - 3 \text{ Washingtons}$$


**3.** This language gives us a lot of flexibility:

- *Before* we count the result of attaching a second collection to a first collection, we can already represent the *result* by using a *specifying-phrase*.
- *After* we have found the result of attaching a second collection to a first collection, we can represent the result by a *number-phrase*.
- Altogether, to summarize the whole *process*, we can **identify** the *specifying phrase* with an **identification-sentence** which we write as follows
  - i. We write the specifying phrase
  - ii. We lengthen the *bar* with an **arrowhead**
  - iii. We write the number-phrase that represents the result.

**EXAMPLE 3.**

i. *Before* we detach from Jack's  Jill's , we can already represent the *result* by the *specifying-phrase*

$$6 \text{ Washingtons } \underline{\hspace{2cm}} - 4 \text{ Washingtons}$$

ii. *After* we have found that the result of detaching from Jack's 

Jill's  is  we can represent the result by

$$4 \text{ Washingtons}$$

iii. Altogether, to summarize the whole *process* with an *identification-sentence* we lengthen the bar with an arrowhead and we write the number-phrase that represents the result of the *detachment*.

$$6 \text{ Washingtons } \underline{\hspace{2cm}} \rightarrow 4 \text{ Washingtons}$$

**4.** Usually, though, we will not write things this way and we only did it above to show how the mathematical language represented the reality. As usual, some of it “goes without saying”:

- In the *specifying phrase*, the *bar* goes without saying
- In the identification sentence, the arrowhead is replaced by the symbol =

subtraction

**EXAMPLE 4.** Instead of writing the specifying phrase

$$6 \text{ Washingtons} \quad \underline{- 2 \text{ Washingtons}}$$

we shall write

$$6 \text{ Washingtons} - 2 \text{ Washingtons}$$

and instead of writing the identification sentence

$$6 \text{ Washingtons} \xrightarrow{- 2 \text{ Washingtons}} 4 \text{ Washingtons}$$

we shall write

$$6 \text{ Washingtons} - 2 \text{ Washingtons} = 4 \text{ Washingtons}$$

### 9.3 Procedure For Subtracting A Number-Phrase

Given two collections, the paper procedure that gives (the *numerator* of) the number-phrase that represents the result of detaching the second collection from the first collection is called **subtraction** and depends on whether the two number-phrases are *basic counting* number-phrases or *decimal* number-phrases.

In order to *subtract* a second *basic* collection from a first *basic* collection, we count *down* from the numerator of the first collection by a length equal to the numerator of the second collection.

There are then two cases depending on whether, when we count *down* from the numerator of the first number-phrase by a length equal to the second numerator, we can complete the count or not.

- If we can complete the count, then the result of the subtraction is just the end-digit.

**EXAMPLE 5.** To subtract Jill's 3 Washingtons from Jack's 7 Washingtons, that is to *identify* the *specifying-phrase*

$$7 \text{ Washingtons} - 3 \text{ Washingtons}$$

- Starting from 7, we count *down* by a length equal to 3:

$$\underline{6, 5, 4}$$

- We can complete the count and the end-digit is 4
- We write the *identification-sentence*:

$$7 \text{ Washingtons} - 3 \text{ Washingtons} = 4 \text{ Washingtons}$$

- In particular, the end-digit can be 0.

**EXAMPLE 6.** To subtract Jill's 5 Washingtons from Jack's 5 Washingtons, that is to *identify* the *specifying-phrase*

$$5 \text{ Washingtons} - 5 \text{ Washingtons}$$

- i. Starting from 5, we count *down* by a length equal to 5:

$$\underline{4, 3, 2, 1, 0}$$

outcast  
incorrect  
strike out

- ii. We can complete the count and the end-digit is 0

- iii. We write the *identification-sentence*:

$$5 \text{ Washingtons} - 5 \text{ Washingtons} = 0 \text{ Washingtons}$$

- If we cannot complete the count, then the subtraction just cannot be done. (At least in this type of situation. We shall see in the next Chapter other situations in which we can end down *past* 0.)

**EXAMPLE 7.** To subtract Jill's 5 **Washingtons** from Jack's 3 **Washingtons**, that is to *identify* the *specifying-phrase*

$$3 \text{ Washingtons} - 5 \text{ Washingtons}$$

But, to *identify* the *specifying-phrase*, we would have to start from 3 and count *down* by a length of 5 but, by the time we got to 0, we would have counted only by a length of 3 and so we cannot complete the count which is as it should be.

## 9.4 Subtraction As Correction

*Subtraction* often comes up after we have done a long string of additions and realized that there is an **outcast**, that is a number-phrase that we shouldn't have added (for whatever reason), so that, as a consequence, the *total* is **incorrect**.

**EXAMPLE 8.** Suppose we had an ice-cream stand and that we had added *sales* as the day went which gave us the following *specifying-phrase*:

$$6 \text{ Washingtons} + 3 \text{ Washingtons} + 7 \text{ Washingtons} + 9 \text{ Washingtons}$$

and that at the end of day we identified the *specifying-phrase* which gave us

$$25 \text{ Washingtons}$$

but that we then realized that 3 **Washingtons** was an *outcast* (it was not a *sale* but money given for some other purpose) with the consequence that 25 **Washingtons** is *incorrect* in that it is not the sum total of the *sales* for the day.

To get the correct total, we have the following two choices for the procedure:

- **Procedure A** would be to **strike out** the *outcast* and redo the entire addition:

**EXAMPLE 9.** In the above example, we *strike out* the *outcast* 3 **Washingtons**

$$6 \text{ Washingtons} + \cancel{3 \text{ Washingtons}} + 7 \text{ Washingtons} + 9 \text{ Washingtons}$$

which gives us

$$22 \text{ Washingtons}$$

cancel out  
adjustment

Of course, since Procedure A is going to involve a lot of unnecessary work redoing all that had been done correctly, it is very inefficient.

- **Procedure B** would be to **cancel out** the *effect* of the outcast in the incorrect total by *subtracting* the outcast from the incorrect total. (Accountants call this “entering an **adjustment**”.)

**EXAMPLE 10.** In the above example, we *subtract* 3 **Washingtons** (the outcast) from 25 **Washingtons** (the incorrect total):

25 **Washingtons** – 3 **Washingtons**

which gives us:

22 **Washingtons**

We now want to *see* that the two procedures *must* give us the same result either way. For that, we place the specifying-phrases in the two procedures side by side and we see that that the remaining number-phrases are the same either way.

**EXAMPLE 11.** In the above example, we have:

6 **Washingtons** + ~~3 **Washingtons**~~ + 7 **Washingtons** + 9 **Washingtons**

and

6 **Washingtons** + ~~3 **Washingtons**~~ + 7 **Washingtons** + 9 **Washingtons** – ~~3 **Washingtons**~~

We see that, either way, the remaining number-phrases are:

6 **Washingtons** + 7 **Washingtons** + 9 **Washingtons**