Chapter 4

Metric-Exponential System[1]

Metric Denominators, 1 – Exponential Numerators, 4.

The Hindu-Arabic worked well in the preceding chapter because we were using it with money and it happens that the real-world items, ***Penny***, ***Dime***, ***One-Dollar-Bill***, ***Ten-Dollar-Bill***, ***Hundred-Dollar-Bill***, ***Thousand-Dollar-Bill*** change at the rate of TEN to ONE and we were able to use the ready-made Place System denominators

Clevelands	Franklins	Hamiltons	Washingtons	Roosevelts	Lincolns

However, in the US, most things, like *length* and *weight*, are not TEN to ONE. The issue then is what to do when dealing with real-world items for which the bundling rates are not TEN to ONE and we do not have ready-made names to fit a Place System.

4.1 Metric Denominators[2]

We will now see a way to create denominators to fit a Base-TEN Place System systematically. These systematic denominators can then be *used*

[1]This Chapter is specially dedicated to these Educologists who breathlessly teach kilometers, meters and centimeters exactly the way they teach miles, feet and inches, that is by way of yet another set of "conversion factors".

[2]Educologists are of course well aware that the law adopted by the National Constituting Assembly of the French Revolution on March 30, 1791 had mandated not only: **i. Decimal prefixes** to create from a primary denominator secondary denominators whose rate of change would match the number of available *shorhands*, namely TEN, but also **ii.** A "natural" definition of the primary units. i.e. neither anthropocentric nor nationalistic.

primary denominator
metric prefix
secondary denominator

just like we used **Roosevelt**, **Washington**, **Hamilton**, **Franklin**, **Cleveland**, and so here we will focus on how these denominators are *created*.

1. Given a **primary denominator**, we will use the following **metric prefixes**

KILO	HECTO	DEKA	–	DECI	CENTI	MILLI

to create **secondary denominators** from the primary denominator.

EXAMPLE 1. If we pick **Franklins** as our primary denominator, then the heading

Cleveland	Franklin	Hamilton	Washington	Roosevelt

becomes

DEKA Franklins	Franklins	DECI Franklins	CENTI Franklins	MILLI Franklins

so that, for instance,

3.2 **Clevelands** becomes 3.2 **DEKAFranklins**,
23.77 **Franklins** remains 23.77 **Franklins**,
7.45 **Hamiltons** becomes 7.45 **DECIFranklins**
0.83 **Washingtons** becomes 0.83 **CENTIFranklins**
53.46 **Roosevelts** becomes 7.45 **MILLIFranklins**.

EXAMPLE 2. If we pick **Washingtons** as our primary denominator, then the heading

Clevelands	Franklins	Hamiltons	Washingtons	Roosevelts

becomes

KILO Washingtons	HECTO Washingtons	DEKA Washingtons	Washingtons	DECI Washingtons

so that, for instance,

3.2 **Clevelands** becomes 3.2 **KILOWashingtons**,
23.77 **Franklins** becomes 23.77 **HECTOWashingtons**,
7.45 **Hamiltons** becomes 7.45 **DEKAWashingtons**.
0.83 **Washingtons** becomes 0.83 **Washingtons**
53.46 **Roosevelts** becomes 7.45 **DECIWashingtons**.

2. The immediate advantage of the metric system is that we can change either the *select denominator* or the *pointed digit* as we did for money in the previous chapter, that is there is a simple interplay between *select denominator* and *pointed digit* that is often expressed by some such "rule" as "when you move the point in one direction, you must move the select denominator in the same direction".

EXAMPLE 3. Given

3.825 **DEKAFranklins**

let us say we wanted the numerator to be 382.5. In other words, we want to "move the base-TEN point two places to the right". To obtain the corresponding *select denominator* according to that "rule", we would then "use as select denominator the denominator two places to the right". This would give us

382.5 **DECIFranklins**

To see that the two are indeed the same, we use a heading:

DEKA Franklins	**Franklins**	**DECI Franklins**	**CENTI Franklins**	**MILLI Franklins**
3	8	2	5	

which shows that if we "move the base-TEN point two places to the right", then the pointed digit is now under the **DECIFranklins** which is therefore the new select denominator.

Note that while we changed the *select denominator* from **DEKAFranklins** to **DECIFranklins**, the *primary denominator* did not change and remained **Franklins**.

3. With all that, while the metric system does allow us to represent collections as large as we want, it does not do so entirely conveniently.

EXAMPLE 4. The metric system allows us to represent NINE MILLION ***Dollars*** "directly", that is by writing

9. **KILOClevelands**

but in order to represent, say, NINETY BILLION ***Dollars*** we can only write

90 000. **KILOClevelands**

which we would not be able to place under a real heading because **Clevelands** is the highest denominator we have and **KILO** the highest metric prefix we have.

? **Clevelands**	? **Clevelands**	? **Clevelands**	? **Clevelands**	**KILO Clevelands**
9	0	0	0	0

As scientists need denominators for larger and larger collections, the General Conference on Weights and Measures introduces, every few years, new prefixes to reduce the number of zeros in number-phrases[3]. However, with a few rare exception, such as **MEGA** and perhaps **GIGA**, "the rest of us" neither knows nor wants to memorize prefixes that we are unlikely ever to use. Instead we proceed as in the next section.

[3]For a list, see `http://en.wikipedia.org/wiki/SI_prefix`

exponential prefix
exponent

4.2 Exponential Numerators

In order not to be limited by the existing metric prefixes and not to have to write large numbers of zeros, scientists use a different kind of prefixes, **exponential prefixes**, which we will now discuss.

1. Back when we introduced the bundling system, we saw that, as the collections got larger, we had to keep bundling so that, together with real-world items, we also got:

i. collections of real-world items,

ii. collections of collections of real-world items,

iii. collections of collections of collections of real-world items,

iv. collections of collections of collections of collections of real-world items,

v. etc.

So, if the real-world items are ***Gizmos*** and the denominator is **Gizmo**, then, since the bundling rate is TEN to ONE, it is rather natural to use:

i. TEN **Gizmo** as a denominator for ***collections of TEN Gizmos***,

ii. TEN TEN **Gizmo** as a denominator for ***collections of TEN collections of TEN Gizmos***,

iii. TEN TEN TEN **Gizmo** as a denominator for ***collections of collections of collections of Gizmos***,

iv. TEN TEN TEN TEN **Gizmo** as a denominator for ***collections of TEN collections of TEN collections of TEN collections of TEN Gizmos***,

v. etc.

From here, it is a short step to introduce the following shorthands:

$\mathbf{TEN}^{1}$ as shorthand for TEN

$\mathbf{TEN}^{2}$ as shorthand for TEN TEN

$\mathbf{TEN}^{3}$ as shorthand for TEN TEN TEN

$\mathbf{TEN}^{4}$ as shorthand for TEN TEN TEN TEN

etc

where the little number on the upper right, called the **exponent**, indicates how many copies of TEN there has to be. And, while we are at it and for the sake of completion, by $\mathbf{TEN}^{0}$ we will mean "no copy of **TEN**"

We could then write:

i. $\mathbf{TEN}^{0}$ **Gizmo** as denominator for ***Gizmos***,

ii. $\mathbf{TEN}^{1}$ **Gizmo** as denominator for ***collections of TEN Gizmos***,

iii. $\mathbf{TEN}^{2}$ **Gizmo** as denominator for ***collections of TEN collections of TEN Gizmos***,

iv. **Ten^3 Gizmo** as denominator for ***collections of TEN collections of TEN collections of TEN Gizmos***,

v. **Ten^4 Gizmo** as denominator for ***collections of TEN collections of TEN collections of TEN collections of TEN Gizmos***,

vi. etc.

2. Given a *primary denominator*, though, not everything works with exponential prefixes as with metric prefixes because we have no way to code the denominators to the right of the primariy denominator.

Example 5. If we pick **Hamiltons** as our primary denominator, then the heading

Cleveland	Franklin	Hamilton	Washington	Roosevelt

becomes

Ten^2 Hamiltons	Ten^1 Hamiltons	Ten^0 Hamiltons	? Hamiltons	? Hamiltons

so that, for instance,

3.2 **Clevelands** becomes 3.2 **Ten^{2}Hamiltons**
23.77 **Franklins** remains 23.77 **Ten^{1}Hamiltons**
7.45 **Hamiltons** becomes 7.45 **Ten^{0}Hamiltons**

but

0.83 **Washingtons** becomes 0.83 ? **Hamiltons**
53.46 **Roosevelts** becomes 7.45 ? **Hamiltons**

Thus, instead of thinking of, say, **Ten^4** as we did above, that is as a shorthand for "4 copies of **Ten**", we will think of the exponent as just indicating "the 4th secondary denominator to the *left* of the primary denominator".
That way, in order also to have "the 4th secondary denominator to the *right* of the primary denominator" we need only some symbol in the exponent to code "left" or "right". It is traditional in this context to use "+" to mean "to the left" and to use "−" to mean "to the right".

Example 6. Say we pick **Hamiltons** as our primary denominator, then the heading that corresponds to

Clevelands	Franklins	Hamiltons	Washingtons	Dimes

in the exponential system involves the following exponential denominators:

Ten^{+2} Hamiltons	Ten^{+1} Hamiltons	Ten^0 Hamiltons	Ten^{-1} Hamiltons	Ten^{-2} Hamiltons

and, for instance,

separator

7.82 **Clevelands** is written 7.82 **Ten**$^{+2}$ **Hamiltons**,
0.081 **Franklins** is written 0.081 **Ten**$^{+1}$ **Hamiltons**,
27.4 **Washingtons** is written 27.4 **Ten**$^{-1}$ **Hamiltons**.

Example 7. For instance, **Ten**$^{+2}$ **Hamiltons** represents the same bill as **Clevelands** because **Clevelands** is the *second* denominator to the *left* of **Hamiltons** while **Washingtons** represents the same bill as **Ten**$^{-1}$ **Hamiltons** because **Washingtons** is the *first* denominator to the *right* of **Hamiltons**.

3. One nice thing about the exponential system is that it helps us keep in mind what the bundling rate is.

Example 8. What would **Two**$^{+3}$ **Gizmo** represent?

4. In reality, the exponential system is not utilized quite as just given. Instead, scientists use a variant in which, instead of **Ten**, they write **x 10** where **x** is a **separator** that is now needed to separate the *numerator* from the *denominator*. (However, as opposed to **Ten**, the prefix **x 10** does *not* encode the exchange rate!)
For instance, picking again **Hamiltons** as our primary denominator, the heading that corresponds to

Clevelands	Franklins	Hamiltons	Washingtons	Dimes

in the exponential system is the heading

x 10^{+2} Hamiltons	x 10^{+1} Hamiltons	x 10^{0} Hamiltons	x 10^{-1} Hamiltons	x 10^{-2} Hamiltons

and, for instance,

7.82 **Clevelands** is written 7.82 **x** 10^{+2} **Hamiltons**,
0.081 **Franklins** is written 0.081 **x** 10^{+1} **Hamiltons**,
27.4 **Washingtons** is written 27.4 **x** 10^{-1} **Hamiltons**.

Note. For rather fascinating illustrations of what this system does, search the web for "Powers of Ten". Here are a few sites worth looking at.
`http://www.youtube.com/watch?v=0fKBhvDjuy0` is the original by the Eames Office.
`http://www.wordwizz.com/pages/10exp0.htm`
http://microcosm.web.cern.ch/microcosm/P10/english/P3.html
http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/