

Chapter 16

Signed Multiplication

Signed-Co-multiplication, 1.

16.1 Effect of Transactions on States: Signed Co-Multiplication

We now want to **extend** the concept of *co-multiplication* to *signed-number-phrases* in order to deal with *actions* and *states*.

1. We begin by looking at the real-world. As before, we want to investigate the *change* in a given state, *gain* or *loss*, that results from a given transaction, “in” or “out” as before but with *two-way collections* of “good” items or “bad” items.

EXAMPLE 1. Consider a store where, for whatever reason best left to the reader’s imagination, collections of apples can either get in or out of the store. Moreover, the collections are really two-way collections in that the apples can be either *good*—inasmuch as they will generate a sales profit—or *bad*—inasmuch as they will have to be disposed of at a cost.

2. We now look at the way we will represent things on paper.

a. To represent collections that can get *in* or *out*, we use *signed number-phrases* and we use a + sign for collections that get *in* and a – sign for *collections* that get *out*.

So, we will represent

- *collections* getting “in” by *positive* number-phrases,
- *collections* getting “out” by *negative* number-phrases,

EXAMPLE 2. In the above example, we would represent

signed co-number-phrase
signed co-multiplication

- a collection of three apples getting *in* the store by the number-phrase +3 Apples
- a collection of three apples getting *out* of the store by the number-phrase −3 Apples

b. To represent unit-values that can be *gains* or *losses*, we use **signed co-number-phrase** and we use a + sign to represent *gains* and a − sign to represent *losses*.

So, we will represent

- the unit-value of “good” items by *positive* co-number-phrases,
- the unit-value of “bad” items by *negative* co-number-phrases,

EXAMPLE 3. In the above example, we would represent

- the unit-value of apples that will generate a sales *profit* of seven cents per apples by the co-number-phrase $+7 \frac{\text{Cents}}{\text{Apple}}$
- the unit-value of apples that will generate a disposal *cost* of seven cents per apple by the co-number-phrase $-7 \frac{\text{Cents}}{\text{Apple}}$

3. Looking at the *effect* that *transactions* (of two-way collections) can have on (money) *states*, that is at the fact that:

- A two-way collection of “good” items getting “in” makes for a “good” change.
- A two-way collection of “good” items getting “out” makes for a “bad” change.
- A two-way collection of “bad” items getting “in” makes for a “bad” change.
- A two-way collection of “bad” items getting “out” makes for a “good” change.

we can now write the procedure for **signed co-multiplication** for which we will use the symbol \otimes :

- i. multiply the *denominators* (with cancellation).
- ii. multiply the *numerators* according to the way gains and losses occur:
 - (+) \otimes (+) gives (+)

EXAMPLE 4.

Three apples get *in* the store.

The apples have a unit-value of seven cents-per-apple *gain*.

The specifying phrase is

We co-multiply

We get a twenty-one cent *gain*.

+3 Apples

$[+3 \text{ Apples}] \otimes \left[+7 \frac{\text{Cents}}{\text{Apple}} \right]$

$[(+3) \otimes (+7)] \left[\frac{\text{Apples}}{\text{Apples}} \times \frac{\text{Cents}}{\text{Apple}} \right]$
= +21 Cents

- (+) \otimes (−) gives (+)

EXAMPLE 5.

Three apples get *in* the store.
 The apples have a unit-value of seven cents-per-apple *loss*.

The specifying phrase is

We co-multiply

We get a twenty-one cent *loss*.

- $(-) \otimes (+)$ gives $(+)$

EXAMPLE 6.

Three apples get *out* of the store.
 The apples have a unit-value of seven cents-per-apple *gain*.

The specifying phrase is

We co-multiply

We get a twenty-one cent *loss*.

- $(-) \otimes (-)$ gives $(+)$

EXAMPLE 7.

Three apples get *out* of the store.
 The apples have a unit-value of seven cents-per-apple *loss*.

The specifying phrase is

We co-multiply

We get a twenty-one cent *gain*.

$$\begin{aligned}
 &+3 \text{ Apples} \\
 &\quad -7 \frac{\text{Cents}}{\text{Apple}} \\
 &[+3 \text{ Apples}] \otimes \left[-7 \frac{\text{Cents}}{\text{Apple}} \right] \\
 &[(+3) \otimes (-7)] \left[\text{Apples} \times \frac{\text{Cents}}{\text{Apple}} \right] \\
 &= -21 \text{ Cents}
 \end{aligned}$$

$$\begin{aligned}
 &-3 \text{ Apples} \\
 &\quad +7 \frac{\text{Cents}}{\text{Apple}} \\
 &[-3 \text{ Apples}] \otimes \left[+7 \frac{\text{Cents}}{\text{Apple}} \right] \\
 &[(-3) \otimes (+7)] \left[\text{Apples} \times \frac{\text{Cents}}{\text{Apple}} \right] \\
 &= -21 \text{ Cents}
 \end{aligned}$$

$$\begin{aligned}
 &-3 \text{ Apples} \\
 &\quad -7 \frac{\text{Cents}}{\text{Apple}} \\
 &[-3 \text{ Apples}] \otimes \left[-7 \frac{\text{Cents}}{\text{Apple}} \right] \\
 &[(-3) \otimes (-7)] \left[\text{Apples} \times \frac{\text{Cents}}{\text{Apple}} \right] \\
 &= +21 \text{ Cents}
 \end{aligned}$$

NOTE. The choice of symbols, + to represent *good* and - to represent *bad*, was not an arbitrary choice because of the way they interact with the symbols for *in* and *out*. We leave it as an exercise for the reader to investigate what happens when other choices are made.

4. Just as with *addition* and *subtraction*, in the case of *co-multiplication* too, we can replace *plain* number-phrases by *positive* number-phrases .

EXAMPLE 8.

