MATH 017 REVIEW I Discussions

[Run: 09/19/2011 at 8:7 Seed: 9744. Order of Checkable Items: List.]

I-1. Given the number-phrases -17.84 **Dollars** and $+6.37$ **Dollars** in that order, which lenient size-comparison sentence is *true*?

Discussion: A *size*-comparison sentence involves only the *size* and not the sign of the signed number-phrases being compared.

We can look at the question from two points of view:

- In a corresponding real-world situation, we have:
  - Jack owes seventeen dollars and eighty-four cents,
  - Jill is owed six dollars and thirty-seven cents

  Since we are *size*-comparing, what matters is only how much is being owed and Jack owes no less than what Jill is owed.
- In the paper representation, we compare only the *sizes* of the signed numbers.

Either way, we end up writing the size-comparison sentence

$$-17.84 \text{ **Dollars** } \textit{is-no-smaller-in-size-than } +6.37 \text{ **Dollars**}$$

I-2. Given the number-phrases -27.32 **Dollars** and -742.33 **Dollars** in that order, which *strict* algebra-comparison sentence is *true*?

Discussion: An *algebra*-comparison sentence involves both the *size* and the *sign* of the signed number-phrases being compared.

We can look at the question from two points of view:

- In a corresponding *real-world situation*, we have:
  - Jack owes twenty-seven dollars and eighty-four cents
  - Jill owes seven hundred, forty-two dollars and thirty-three cents

  Since we are *algebra*-comparing, what matters is that, since Jack *owes less* than Jill, Jack is *better off* than Jill.
- In the *paper representation*, any two *negative* number-phrase *algebra-compare* the way *opposite* to the way they *size-compare*.

Either way, we end up writing the algebra-comparison sentence

$$-27.32 \text{ **Dollars** } > -742.33 \text{ **Dollars**}$$

I-3. The single action that gives the same result as a twenty-three dollars and fifty-two cents deposit followed by a sixty-eight dollars and seventy-six cents withdrawal is represented by what signed number-phrase?

Discussion: *Depositing* money and *withdrawing* money are real-world *actions* that we represent on paper by *signed number-phrases. Following* a first action by a second action is represented on paper by the *addition* of the second number-phrases to the first number-phrase.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - Jack deposits twenty-three dollars and thirty-two cents
  - Jill withdraws sixty eight dollars and thirty-three cents

  Since Jill's action is more in size than Jack's action, she is in fact withdrawing the twenty-three dollars and thirty-two cents that Jack had deposited and another forty-five dollars and twenty-four cents.
- In the *paper representation*, we write the specifying number-phrase

  $$+23.52 \textsf{ Dollars} \oplus -68.76 \textsf{ Dollars}$$

  and then THEOREM 1 says that since the number-phrases are *opposite* in sign,
  - We get the *sign* of the result by taking the sign of the signed number-phrase whose *size* is *larger*,
  - We get the *size* of the result by *subtracting* the smaller size from the larger size.

Either way, we end up writing the signed number-phrase

$$-45.24 \textsf{ Dollars}$$

I-4. You thought your balance was one hundred seventy-two dollars and fifty-seven cents in the black but you just found out that a twelve dollars and fifty-six cents check you had deposited bounced. What is the signed number-phrase that represents your new balance?

Discussion: *Removing* a deposit or *removing* a withdrawal is a real-world *action* that is represented on paper by a *subtraction.*

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - You thought the balance was one hundred seventy-two dollars and fifty-seven cents in the black
  - but this balance included a twelve dollars and fifty-six cents check

Since the check bounced, the balance is actually twelve dollars and fifty-six cents less than you thought, that is one hundred sixty dollars and one cent in the black.

- In the *paper representation*, we write the specifying-phrase

$$+172.57 \textsf{ Dollars} \ominus +12.56 \textsf{ Dollars}$$

which we identify by *adding the opposite of* the second number-phrase to the first number-phrase

$$+172.57 \textsf{ Dollars} \oplus -12.56 \textsf{ Dollars}$$

Either way, we end up writing the signed number-phrase

$$+160.01 \textsf{ Dollars}$$

I-5. You thought your balance was one hundred seventy-two dollars and fifty-seven cents in the red but you just found out that an unjustified twelve dollars and fifty-six cents charge has been removed. What is the signed number-phrase that represents your new balance?

Discussion: *Removing* a deposit or *removing* a withdrawal is a real-world *action* that is represented on paper by a *subtraction*.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - You thought the balance was one hundred seventy-two dollars and fifty-seven cents in the red
  - but this balance included a twelve dollars and fifty-six cents charge

  Since the charge was removed, the balance is actually twelve dollars and fifty-six cents more than you thought, that is one hundred sixty dollars and one cent in the red.
- In the *paper representation*, we write the specifying-phrase

$$-172.57 \textsf{ Dollars} \ominus -12.56 \textsf{ Dollars}$$

which we identify by *adding the opposite of* the second number-phrase to the first number-phrase

$$-172.57 \textsf{ Dollars} \oplus +12.56 \textsf{ Dollars}$$

Either way, we end up writing the signed number-phrase

$$-160.01 \textsf{ Dollars}$$

I-6. On Monday your balance was three hundred thirty-two dollars and seventy one cents in the red and on Thursday your balance was seventy-four dollars and forty-six cents in the red. What is the signed number-phrase that represents the change in your balance from Monday to Thursday?

Discussion: The *change* is the single action on the initial state that results in the final state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - on Monday your balance was three hundred thirty-two dollars and seventy one cents in the red
  - on Thursday your balance was seventy-four dollars and forty-six dollars in the red

  Since, while still in the red, the balance has gone *down in size*, from three hundred thirty-two dollars and seventy-one cents to seventy-four dollars and forty-six dollars, this means that the action must have been a gain of two hundred fifty-eight dollars and twenty-five cents.
- In the *paper representation*, THEOREM 2 says that the *change* from an initial state to a final state is equal to the final state *ominus* the initial state. So we write the specifying-phrase
  $$-74.46 \textsf{ Dollars} \ominus -332.71 \textsf{ Dollars}$$
  that is
  $$-74.46 \textsf{ Dollars} \oplus +332.71 \textsf{ Dollars}$$

Either way, we end up writing

$$-258.25 \textsf{ Dollars}$$

I-7. On Tuesday your balance was six hundred three dollars and twenty-eight cents in the red and on Friday your balance was fifty-six dollars and three cents in the black. What is the signed number-phrase that represents the change in your balance from Tuesday to Friday?

Discussion: The *change* is the single action on the initial state that results in the final state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - on Tuesday your balance was six hundred three dollars and twenty-eight cents in the red
  - on Friday your balance was fifty-six dollars and three cents in the black

  Since your balance has gone from the red to the black, from six hundred three dollars and twenty-eight cents in the red to fifty-six dollars and three cents in the black, this means that the action must have been a gain of six hundred three dollars and twenty-eight cents plus fty-six dollars and three cents, that is a gain of six hundred fifty-nine dollars and thirty one cents.

- In the *paper representation*, THEOREM 2 says that the *change* from an initial state to a final state is equal to the final state *ominus* the initial state. So we write the specifying-phrase

  $$+56.03 \textsf{ Dollars} \ominus -603.28 \textsf{ Dollars}$$

  that is

  $$+56.03 \textsf{ Dollars} \oplus +603.28 \textsf{ Dollars}$$

Either way, we end up writing

$$+659.31 \textsf{ Dollars}$$

I-8. Your balance was seventy-six dollars and thirty-eight cents in the red and you made an eight hundred seventy-six dollars and eleven cents withdrawal. What is the signed number-phrase that represents your new balance?

Discussion: The *final* state is the result of the *action* on the *initial* state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - The initial state of your account was seventy-six dollars and thirty-eight cents in the red
  - The action on this initial state was an eight hundred and seventy-six dollars and eleven cents withdrawal.

  Since your are *withdrawing* money from an account that was already in the *red*, the eight hundred and seventy-six dollars and eleven cents add to the seventy-six dollars and thirty-eight cents to give a final balance of *nine hundred fifty-two dollars and forty-nine cents in the black*.
- In the *paper representation*, we write the signed specifying-phrase

  $$-76.38 \textsf{ Dollars} \oplus -876.11 \textsf{ Dollars}$$

  and we identify it.

Either way, we end up writing

$$+952.49 \textsf{ Dollars}$$

I-9. Your balance was seventy-six dollars and thirty-eight cents in the black and you made an eight hundred seventy-six dollars and eleven cents withdrawal. What is the signed number-phrase that represents your new balance?

Discussion: The *final* state is the result of the *action* on the *initial* state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - The initial state of your account was seventy-six dollars and thirty-eight cents in the black
  - The action on this initial state was an eight hundred and seventy-six dollars and eleven cents withdrawal.

  Since your are *withdrawing* more money than was in the account, the eight hundred and seventy-six dollars and eleven cents break down to the seventy-six dollars and thirty-eight cents that were in the account and the remainder that gives a final balance of *seven hundred ninety-nine dollars and seventy-three cents in the red.*
- In the *paper representation*, we write the signed specifying-phrase

  $$+76.38 \textsf{ Dollars} \oplus -876.11 \textsf{ Dollars}$$

  and we identify it.

Either way, we end up writing

$$-799.73 \textsf{ Dollars}$$

I-10. Your balance was seventy-six dollars and thirty-eight cents in the red and you made an eight hundred seventy-six dollars and eleven cents deposit. What is the signed number-phrase that represents your new balance?

Discussion: The *final* state is the result of the *action* on the *initial* state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - The initial state of your account was seventy-six dollars and thirty-eight cents in the red
  - The action on this initial state was an eight hundred and seventy-six dollars and eleven cents deposit.

  Since your are *depositing* money from an account that was already in the *red*, the eight hundred and seventy-six dollars and eleven cents first go to the seventy-six dollars and thirty-eight cents in the red to give a final balance of *seven hundred ninety-nine dollars and seventy-three cents in the black*

- In the *paper representation*, we write the signed specifying-phrase

  $$-76.38 \textsf{ Dollars} \oplus +876.11 \textsf{ Dollars}$$

  and we identify it.

Either way, we end up writing

$$+799.73 \textsf{ Dollars}$$

I-11. Your balance was seventy-six dollars and thirty-eight cents in the black and you made an eight hundred seventy-six dollars and eleven cents deposit. What is the signed number-phrase that represents your new balance?

Discussion: The *final* state is the result of the *action* on the *initial* state.

We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - The initial state of your account was seventy-six dollars and thirty-eight cents in the black
  - The action on this initial state was an eight hundred and seventy-six dollars and eleven cents deposit.

  Since your are *depositing* money on an account that was already in the *black*, the eight hundred and seventy-six dollars and eleven cents add to the seventy-six dollars and thirty-eight cents to give a final balance of *nine hundred fifty-two dollars and forty-nine cents in the black*.
- In the *paper representation*, we write the signed specifying-phrase

  $$+76.38 \textsf{ Dollars} \oplus +876.11 \textsf{ Dollars}$$

  and we identify it.

Either way, we end up writing

$$+952.49 \textsf{ Dollars}$$

I-12. What is the *distance between* -332.71 **Dollars** *and* -74.46 **Dollars**

Discussion: The *distance* between two signed number-phrases is the *size* of the *change* from either one to the other.

We obtain the *change from* -332.71 **Dollars** *to* -74.46 **Dollars** by subtracting the first signed number-phrase from the second number-phrase:

$$-74.46 \text{ Dollars} \ominus -332.71 \text{ Dollars}$$

that is

$$-74.46 \text{ Dollars} \oplus +332.71 \text{ Dollars}$$

which gives the change

$$+258.25 \text{ Dollars}$$

whose *size* is

$$258.25 \text{ Dollars}$$

If we compute the *change from* -74.46 **Dollars** *to* -332.71 **Dollars**, we get the *opposite* change:

$$-258.25 \text{ Dollars}$$

whose *size*, though, is still the same:

$$258.25 \text{ Dollars}$$

I-13. What is the *distance between* -332.71 **Dollars** *and* $+74.46$ **Dollars**

Discussion: The *distance* between two signed number-phrases is the *size* of the *change* from either one to the other.

We obtain the *change from* -332.71 **Dollars** *to* $+74.46$ **Dollars** by subtracting the first signed number-phrase from the second number-phrase:

$$+74.46 \text{ Dollars} \ominus -332.71 \text{ Dollars}$$

that is

$$+74.46 \text{ Dollars} \oplus +332.71 \text{ Dollars}$$

which gives the change

$$+407.17 \text{ Dollars}$$

whose *size* is still the same

$$407.17 \text{ Dollars}$$

If we compute the *change from* $+74.46$ **Dollars** *to* -332.71 **Dollars**, we get the *opposite* change:

$$-407.17 \text{ Dollars}$$

whose *size*, though, is the same:

$$407.17 \text{ Dollars}$$

I-14. Plot the number phrase(s) that is/are at a 3 **Dollars** distance from -1 **Dollars**.

Discussion: The *distance* between two signed number-phrases is the *size* of the *change* from either one to the other.

We can proceed in either one of two ways:

- From the *algebraic* viewpoint, we say that, since the *distance* of the *final* signed-number-phrase from the *initial* signed number-phrase -1 **Dollars** is required to be 3 **Dollars**, then the *change* can be either
  - ⋆ *positive*, that is $+3$ **Dollars**, and the *final* signed number-phrase is:

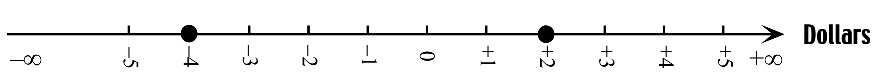
$$\begin{aligned}\text{Initial} \oplus \textit{positive} \text{ change} &= -1 \text{ Dollars} \oplus +3 \text{ Dollars} \\ &= +2 \text{ Dollars}\end{aligned}$$

  or
  - ⋆ *negative*, that is -3 **Dollars**, and the *final* signed number-phrase is:

$$\begin{aligned}\text{Initial} \oplus \textit{negative} \text{ change} &= -1 \text{ Dollars} \oplus -3 \text{ Dollars} \\ &= -4 \text{ Dollars}\end{aligned}$$

  both of which we then plot.
- From the *graphic* viewpoint, we count from the given initial number-phrase -1 **Dollars** a distance of 3 **Dollars** in both directions.

Either way, we end up with



I-15. Plot the number phrase(s) that is/are at a 2 **Dollars** distance from $+3$ **Dollars**.

Discussion: The *distance* between two signed number-phrases is the *size* of the *change* from either one to the other.

We can proceed in either one of two ways:

- From the *algebraic* viewpoint, we say that, since the *distance* of the *final* signed-number-phrase from the *initial* signed number-phrase $+3$ **Dollars** is required to be 2 **Dollars**, then the *change* can be either
  - ⋆ *positive*, that is $+2$ **Dollars**, and the *final* signed number-phrase is:

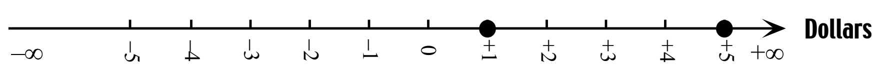
$$\begin{aligned}\text{Initial} \oplus \textit{positive} \text{ change} &= +3 \textsf{ Dollars} \oplus +2 \textsf{ Dollars}\\ &= +5 \textsf{ Dollars}\end{aligned}$$

  or
  - ⋆ *negative*, that is -2 **Dollars**, and the *final* signed number-phrase is:

$$\begin{aligned}\text{Initial} \oplus \textit{negative} \text{ change} &= +3 \textsf{ Dollars} \oplus -2 \textsf{ Dollars}\\ &= +1 \textsf{ Dollars}\end{aligned}$$

  both of which we then plot.
- From the *graphic* viewpoint, we count from the given initial number-phrase $+3$ **Dollars** a distance of 2 **Dollars** in both directions.

Either way, we end up with



I-16. Identify the specifying-phrase $(-116.72) \oplus (-54.07)$

Discussion: We use, from Chapter 5, the following

THEOREM 1. *To add signed-numerators:*

- *When the two signed number-phrases have the* same *sign,*
  - *We get the* sign *of the result by taking the common sign*
  - *We get the* size *of the result by* adding *the two sizes.*
- *When the two signed number-phrase have* opposite *signs, we must first* compare *the* sizes *of the two signed number-phrases and then*

   - *We get the* sign *of the result by taking the sign of the signed number-phrase whose* size *is* larger*,*
   - *We get the* size *of the result by* subtracting *the smaller size from the larger size.*

Since, here, the two signed numerators have the *same* sign $-$,

- we get the *sign* of the result by taking the common sign $-$
- we get the *size* of the result by adding the two sizes: $116.72 + 54.07$

Altogether, we have identified the specifying-phrase $(-116.72) \oplus (-54.07)$ as

$$-170.79$$

I-17. Identify the specifying-phrase $-395.82 \oplus +47.93$

Discussion: We use, from Chapter 5, the following

THEOREM 1. *To add signed-numerators:*

- *When the two signed number-phrases have the* same *sign,*
   - *We get the* sign *of the result by taking the common sign*
   - *We get the* size *of the result by* adding *the two sizes.*
- *When the two signed number-phrase have* opposite *signs, we must first* compare *the* sizes *of the two signed number-phrases and then*
   - *We get the* sign *of the result by taking the sign of the signed number-phrase whose* size *is* larger*,*
   - *We get the* size *of the result by* subtracting *the smaller size from the larger size.*

Since, here, the two signed numerators have *opposite* signs, we must compare the *sizes* of the two numerators.

- we get the *sign* of the result by taking the sign of -395.82 since it is larger in size than $+47.93$.
- we get the *size* of the result by subtracting the smaller size from the larger size: $395.82 - 47.93$

Altogether, we have identified the specifying-phrase $-395.82 \oplus +47.93$ as

$$-347.89$$

I-18. Identify the specifying-phrase $+496.81 \ominus -52.59$

Discussion: To *subtract* a signed number-phrase means to *add the opposite* of this signed number-phrase.

So,

$$\begin{aligned} +496.81 \ominus -52.59 &= +496.81 \oplus +52.59 \\ &= +549.4 \end{aligned}$$

I-19. Identify the specifying-phrase $+728.22 \ominus +76.29$

Discussion: To *subtract* a signed number-phrase means to *add the opposite* of this signed number-phrase.

So,

$$\begin{aligned} +728.22 \ominus +76.29 &= +728.22 \oplus -76.29 \\ &= +651.93 \end{aligned}$$

I-20. Identify $2 - 1 + 4 - 1 - 3 + 5 - 3 - 2 + 1 + 6 - 1 + 5 + 2$

Discussion:

i. The symbol $\oplus$ goes without saying,

ii. The symbols $+$ and $-$ are the signs of the signed numerators,

iii. If the first numerator has no sign, the sign $+$ goes without saying.

$2 - 1 + 4 - 1 - 3 + 5 - 3 - 2 + 1 + 6 - 1 + 5 + 2$

$$\underbrace{+2 \oplus -1}$$
$$\underbrace{+1 \oplus +4}$$
$$\underbrace{-3 \oplus -1}$$
$$\underbrace{-4 \oplus -3}$$
$$\underbrace{-7 \oplus +5}$$
$$\underbrace{-2 \oplus -3}$$
$$\underbrace{-5 \oplus -2}$$
$$\underbrace{-7 \oplus +1}$$
$$\underbrace{-6 \oplus +6}$$
$$\underbrace{0 \oplus -1}$$
$$\underbrace{-1 \oplus +5}$$
$$\underbrace{+4 \oplus +2}$$
$$+6$$

I-21. Identify $-1-1+2+2-3-3+4+4-5-5+6+6$

Discussion:

i. The symbol $\oplus$ goes without saying,

ii. The symbols $+$ and $-$ are the signs of the signed numerators,

iii. If the first numerator has no sign, the sign $+$ goes without saying.

$-1-1+2+2-3-3+4+4-5-5+6+6$

$$\underbrace{-1\oplus-1}$$
$$\underbrace{-2\oplus+2}$$
$$\underbrace{0\oplus+2}$$
$$\underbrace{+2\oplus-3}$$
$$\underbrace{-1\oplus-3}$$
$$\underbrace{-4\oplus+4}$$
$$\underbrace{0\oplus+4}$$
$$\underbrace{+4\oplus-5}$$
$$\underbrace{-1\oplus-5}$$
$$\underbrace{-6\oplus+6}$$
$$\underbrace{0\oplus+6}$$
$$+6$$

I-22. What should you *add* to -3 **Dollars** in order to get $+7$ **Dollars**?

Discussion: A simple way to do this is first to get 0 **Dollars**:

i. Starting with -3 **Dollars** and in order to get 0 **Dollars** we must add the opposite of -3 **Dollars**, that is $+3$ **Dollars**.

ii. Starting now with 0 **Dollars** and in order to get $+7$ **Dollars** we must add $+7$ **Dollars**.

So, altogether, starting with -3 **Dollars** and in order to get $+7$ **Dollars** we must add:

$$+3 \textsf{ Dollars} \oplus +7 \textsf{ Dollars} = +10 \textsf{ Dollars}$$

We check that

$$-3 \textsf{ Dollars} \oplus +10 \textsf{ Dollars} = +7 \textsf{ Dollars}$$

I-23. What should you *subract* from -3 **Dollars** in order to get $+7$ **Dollars**?

Discussion: To ask:

What should we subtract from -3 **Dollars** in order to get $+7$ **Dollars**

is the same as to ask:

What should we add to $+7$ **Dollars** in order to get -3 **Dollars**

So then,

i. Starting with $+7$ **Dollars** and in order to get 0 **Dollars** we must add the opposite of $+7$ **Dollars**, that is -7 **Dollars**.

ii. Starting now with 0 **Dollars** and in order to get -3 **Dollars** we must add -3 **Dollars**.

So, altogether, starting with $+7$ **Dollars** and in order to get -3 **Dollars** we must add:

$$-7\ \textbf{Dollars} \oplus -3\ \textbf{Dollars} = -10\ \textbf{Dollars}$$

We check that

$$\begin{aligned} -3\ \textbf{Dollars} \ominus -10\ \textbf{Dollars} &= -3\ \textbf{Dollars} \oplus +10\ \textbf{Dollars} \\ &= +7\ \textbf{Dollars} \end{aligned}$$

I-24. Identify the specifying-phrase $[+4\ \textbf{Apples}] \times \left[-2\ \frac{\textbf{Dimes}}{\textbf{Apple}}\right]$

Discussion: We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - We have four apples appearing into the warehouse
  - Each of these apples are bad apples and will cost two dimes per apple to get rid of.

  Altogether then, this is going to cost eight dimes to the business.
- In the *paper representation*, we *co-multiply*:

  i. we multiply the *denominators* (with cancellation):
  $$\cancel{\textbf{Apples}} \times \frac{\textbf{Dimes}}{\cancel{\textbf{Apple}}} = \textbf{Dimes}$$
  ii. we multiply the sizes of the numerators
  $$4 \times 2 = 8$$
  iii. we multiply the signs of the numerators
  $$(+) \otimes (-) \text{ gives } (-)$$

Either way, we have identified the specifying-phrase $[+4\ \textbf{Apples}] \times \left[-2\ \frac{\textbf{Dimes}}{\textbf{Apple}}\right]$ as

-8 **Dimes**

I-25. Identify the specifying-phrase $[-5 \text{ Carrots}] \times \left[+7 \ \frac{\text{Cents}}{\text{Carrot}}\right]$

Discussion: We can look at the question from two points of view:

- In a corresponding *real-world situation*,
  - We have five carrots disappearing from the warehouse
  - These carrots were bad carrots and would have cost seven cents per carrot to get rid of.

  Altogether then, this is going to be a gain of thirty-five cents for the business.
- In the *paper representation*, we *co-multiply*:

  i. we multiply the *denominators* (with cancellation):
  $$\cancel{\text{Carrots}} \times \frac{\text{Cents}}{\cancel{\text{Carrot}}} = \text{Cents}$$
  ii. we multiply the sizes of the numerators
  $$5 \times 7 = 35$$
  iii. we multiply the signs of the numerators
  $$(-) \otimes (-) \text{ gives } (+)$$

Either way, we have identified the specifying-phrase $[-5 \text{ Carrots}] \times \left[-7 \ \frac{\text{Cents}}{\text{Carrot}}\right]$ as

$+35$ **Cents**