Notes from the Mathematical Underground

Edited by

Alain Schremmer.
Mathematics Department, Community College of Philadelphia,
1700 Spring Garden Street
Philadelphia, PA 19130

Since this is a new column, a *caveat* might be in order. As we all know, there currently is a "crisis" in mathematics education. (As has been the case, roughly, as far back as I can remember, which is, alas, quite far enough.) Since the latter involves mathematics, faculty and students, not necessarily in that order, this is where its origin must be sought and this is what I would like to do here. By definition however, faculty are above suspicion or, at least, will be in this column. Students being *the* given of the situation and, in any case, presumably not part of the readership, any discussion of how they *ought* to be would be futile and I will leave them mostly alone and not discuss pedagogy. This leaves mathematics and this is what I intend to discuss: No "how to do it", no sugar-coating by way "applications" or "math history" or whatever, no "high tech" religion. But mathematics does not exist in a vacuum and my distinguished colleagues will, of necessity, have to be considered as *parameters* in the equation.

It might have occurred to the reader that I left textbooks out of this equation and it is indeed fashionable in academic circles to deplore the state of the textbook art (Except among authors of course. See, for instance, Anton (1991)) and blame it for much of the educational fiasco. As publishers, though, are fond to point out, it is faculty who design the courses, who order the textbooks and who write them. Moreover, when they wish to be nasty, they are quite prone to listing all the non-conformist texts on which they say they lost their corporate shirt.

Which brings me, precisely, to the main issue that I intend to pursue here, that of the alternatives to the *mathematical* underpinnings of our teaching. One way then in which I would like to do this is to discuss textbooks that didn't make it as mainstream texts because, presumably, they were "too different". Here, I mainly think of calculus texts such as those by Levi, Keisler, Strang, Flanigan-Kazdan, Freed, etc. I would also like to discuss ideas that briefly appeared in texts but were dropped in subsequent editions—when there was one, such as Munroe's definition of variables or Gillman-McDowell's definition of the integral. There are also very simple ideas, such as Lang's treatment of the transcendental functions or that of Finney-Ostbey, that appear in more advanced texts but which, somehow, never made it to "elementary" textbooks .

To give a more general yet concrete example of what I have in mind, a glance at any Arithmetic text will show why the Metric System is not catching in this country: It is presented exactly in the same manner as the now almost defunct English system, namely as just another collection of units with conversion factors to be memorized. In other words, a system it is not. The key to the Metric System is that it complements the Decimal System by providing *units* that behave decimally. To take money as our *model*—it is, after all, the only metric entity in the U. S., we have the following *units*: **Cleveland**, **Franklin**, **Hamilton**, **Washington**, **Dime**, **Penny**, **Milly** each worth ten of the next one. The idea is that one is not allowed to hold more that nine objects of a kind because we could not symbolize such a holding with ten digits. This requires the notion of *exchange*.

> **EXAMPLE 1.** Say we hold, on the one hand, **5 Hamiltons**, **6 Washingtons**, **4 Dimes** and, on the other hand, **7 Washingtons**, **8 Dimes**. Adding the dimes, we get twelve dimes which we are not allowed to hold so that we must exchange ten of them for one Washington. We now have fourteen Washington, ten of which we must exchange for one Hamilton. Altogether, we now have: **6 Hamiltons**, **4 Washingtons**, **2 Dimes**.

> **EXAMPLE 2.** Say we hold **3 Hamiltons**, **0 Washingtons**, **5 Dimes** out of which we must pay **7 Dimes**. Normally, to get dimes, we would exchange one Washington for ten dimes but we do not have any Washington so that we must first exchange one Hamilton for ten Washingtons and then one Washington for ten dimes so that we now have fifteen dimes from which we can now pay the seven dimes that we owe and which leaves us with eight dimes. Altogether, we are left with: **2 Hamiltons**, **9 Washingtons**, **8 Dimes**.

> **EXAMPLE 3.** Say three robbers stick up the bank of a one-bank town and run away with **2 Clevelands**, **9 Franklins**, **0 Hamilton**, **5 Washingtons**. Dividing the loot, each will only get **0 Cleveland**, **3 Franklins**, **0 Hamilton**, **1 Washington** since they can hardly return to the bank to *exchange* the remaining **2 Cleveland**, **0 Franklin**, **0 Hamilton**, **2 Washington** for further division. The usual algorithm is based on the—usually unstated—assumption that changing facilities are available.

Seen in this light then, all the metric system does is to substitute **Kilo$**, **Hecto$**, **Deka$**, **$**, **Deci$**, **Centi$**, **Milli$** for the above units. Observe that adhering tightly to the mathematics of the matter makes much more sense than speaking of "carry over" or "borrowing". Moreover, this allows for a much more compact notation and a great deal of flexibility: Given a holding, it is easy to describe it in terms of any unit one wishes.

> **EXAMPLE 3.** Instead of writing 3 **Kilo$**, 4 **Hecto$**, 5 **Deka$**, 6 **$**, 7 **Deci$**, 8 **Centi$** and assuming we wanted to think in terms of **Hecto$**, we would write 34.5678 **Hecto$** where the purpose of the decimal point is to *point at* the digit that corresponds to the **Hecto$**. If we wanted to think in terms of **Deci$**, we would just move the point accordingly: 34567.8 **Deci$**.

While this is called *unit conversion*, observe that it does not entail any exchange in the above sense.

More generally, as two-year College faculty, we are caught between the rock of what our students are and the hard place of most of the curriculum being driven by four-year institutions. While most of us tend to play it safe, that is conventionally, there is an Underground that tends to rebel and do things somewhat differently and if this Journal, just like any other professional journal, must reflect what, in the main, *is*, it is to the glory of the AMATYTC Review—and of its Editor, Joe Browne, that it would offer some room to those of us who think differently in mathematical matters, presumably on the basis of there being much that we can learn from this Underground as to what *could be*. As for my own intentions concerning this column then, I mainly want to bring up such specific matters as above and argue, with or against whomever cares to write me, *why* we insist on teaching the usual nonsense. I would very much hope for the Underground to become the heart of this column and this column is open not just to the Underground but also to all those who wish to argue with the Underground.

However, having once dabbled in "generalized abstract nonsense", I would like also to discuss less directly mathematical things not usually discussed in so-called professional journals but which I nevertheless deem very relevant, both to our profession and to my purpose in this column. For instance, we declare that mathematics is useful but what do we mean by that? It is often said that this country lacks mathematicians or, at least, that a good background in mathematics helps in getting a good job but, given the number of PhDs in mathematics that did *not* get a permanent position these last few years, I have my doubts as to the first saying and, on logical grounds, I wonder exactly *why* the second saying should be true at all as I shall argue in a future column.

I hold that I learned to read because I felt that reading would afford me pleasure, because it allowed me to escape the real world, just as playing chess would, and not because I needed to be able to read owner's manuals. Similarly, I did not learn what I know of mathematics because it allowed me to find the profit a farmer makes by enclosing his field at so many francs a meter of barbed wire, etc. (This traces me, doesn't it?) It was, partly in spite of my teachers, even though they taught *mathematics*, not cooking by numbers, because I fell in love with mathematics and wanted to play with it, this even though I knew very early on that this love would never be requited and that I would never be able to prove a new lemma or even a new theorem. I hold that we do a disservice to our students, to ourselves and to mathematics, not necessarily in that order, in thinking that anybody needs a contingent reason for studying mathematics.

Reference

Anton, H. (1991). In Defense of the Fat Calculus Text. *UME Trends*, . 2(6), 1.